Spherical mirrors

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

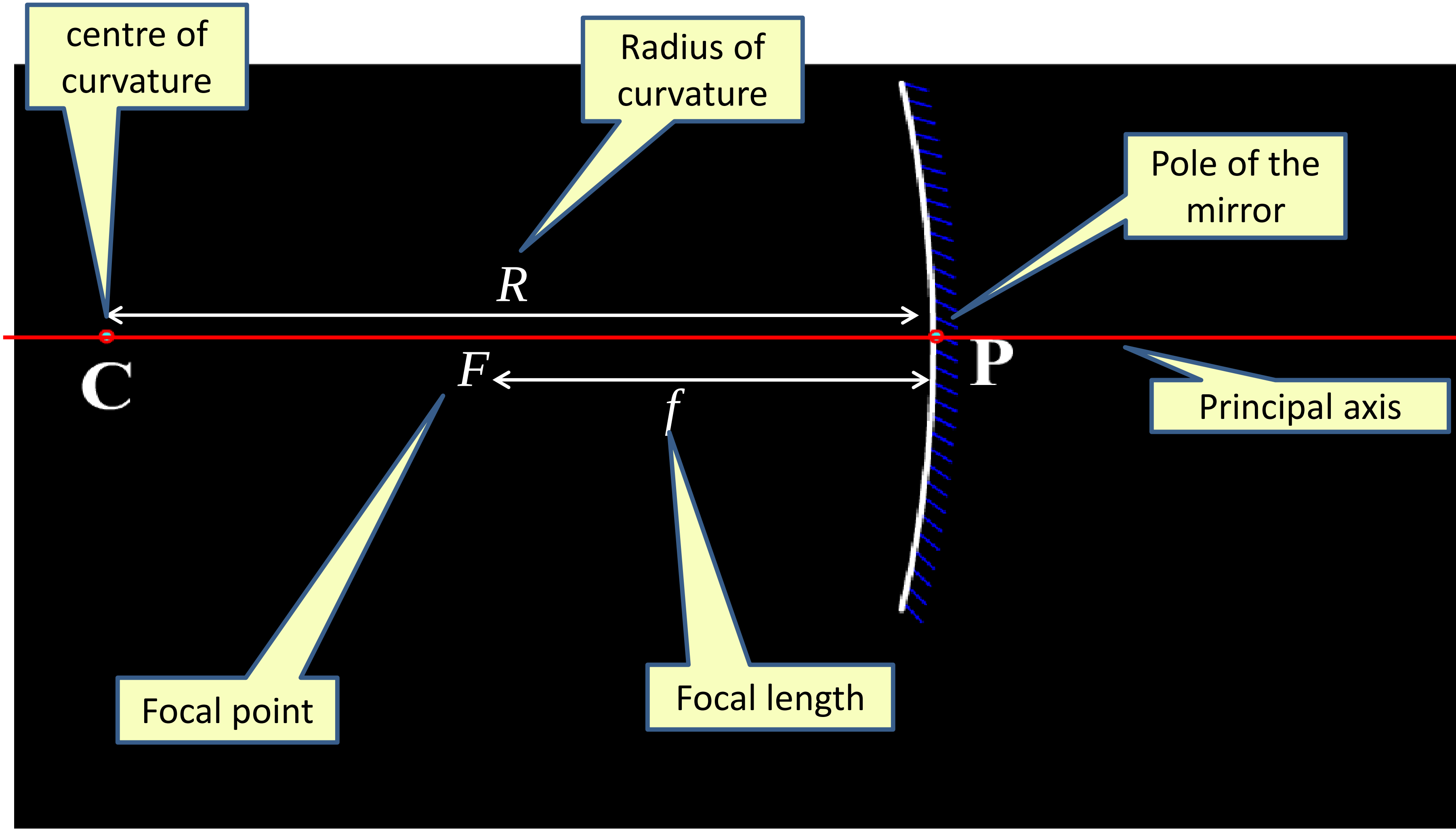
Spherical mirrors

Terminology and parameters

- Centre of curvature (C) : Centre of the imaginary sphere out of which the mirror may be assumed to be made.
- Pole of the mirror (P) : The geometric centre of the mirror may be considered as its pole.
- Principal axis : The line joining the centre of curvature and the pole of the mirror
- Radius of curvature (R) : It is the distance between the pole of the mirror and the centre of curvature.
- Principal focus (F) : The point at which the reflected beam comes to focus, if the incident is parallel to the principal axis.
- Focal length (f) : The distance between the pole and the focal point.

Spherical mirrors

Terminology and parameters



Spherical mirrors

Rules* of construction of images

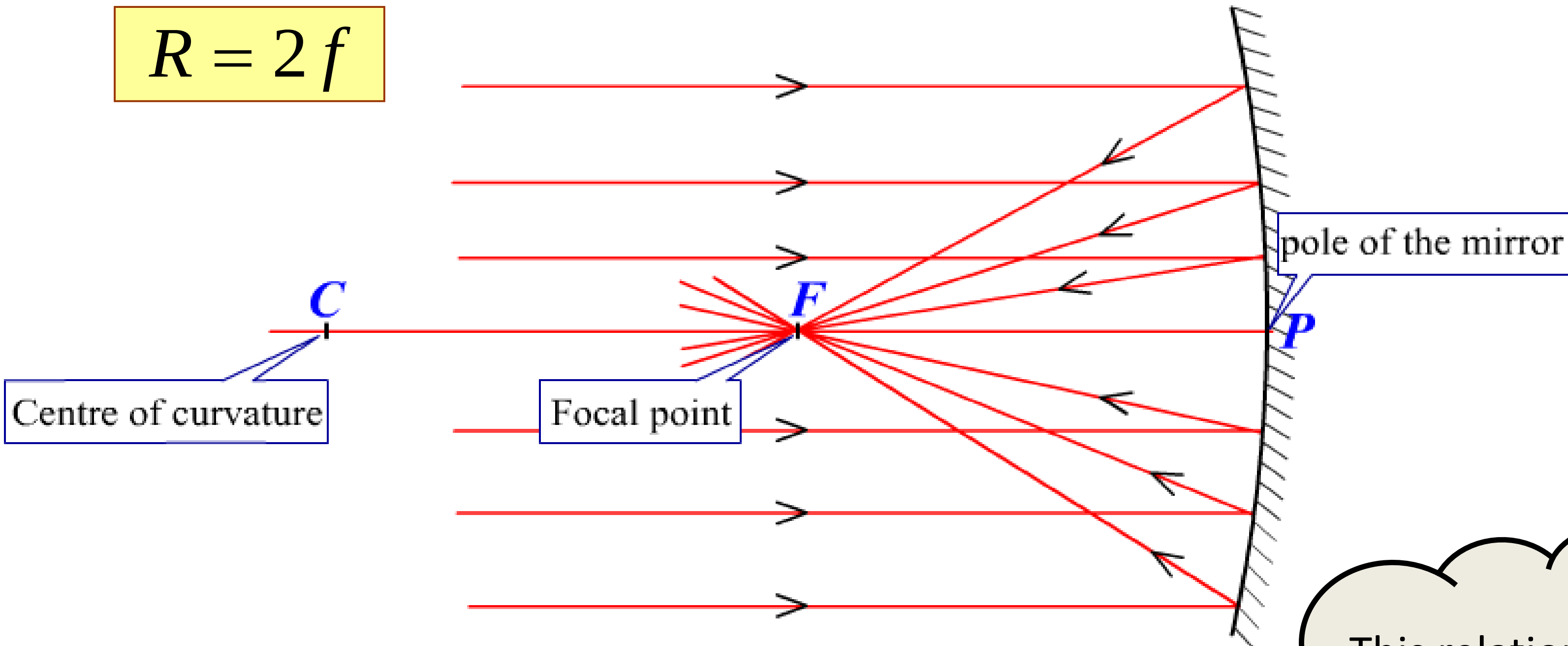
- ❑ An incident ray parallel to the principal axis, upon reflection, passes through the focal point of concave mirror.
- ❑ An incident ray passing through the focal point, upon reflection, becomes parallel to the principal axis
- ❑ An incident ray passing through the centre of curvature is reflected along the same path.
- ❑ A ray incident on the pole of the mirror is reflected making the same angle w.r.t. the principal axis.

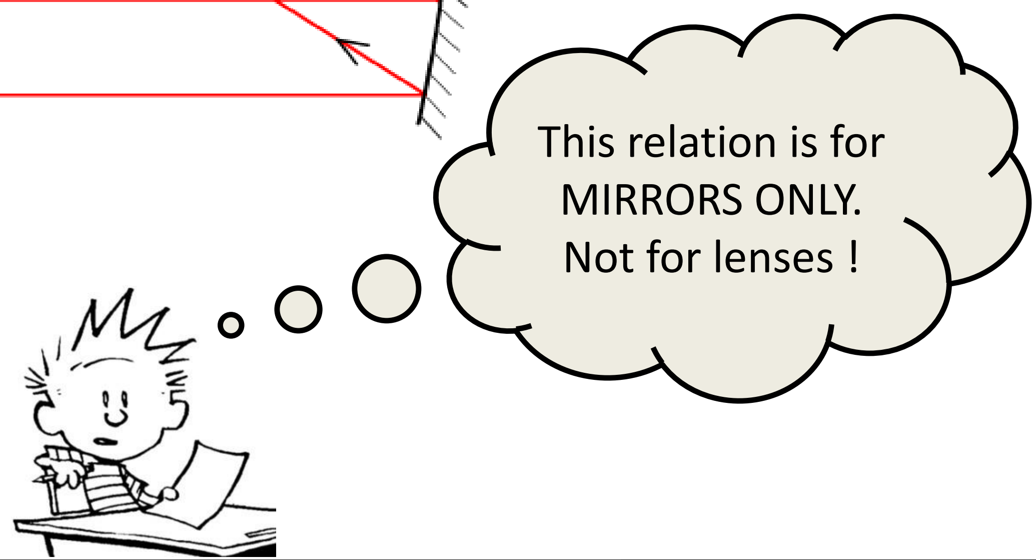
- Though called *rules* , the above mentioned choice of rays is just a matter of convenience.
- Any ray incident of a mirror follows the laws of reflection. Taking rays other than the ones mentioned above makes it relatively difficult to trace the reflected rays and obtain the location of image.

Spherical mirrors

In case of mirrors, radius of curvature is always twice the focal length.

$$\boxed{R = 2f}$$



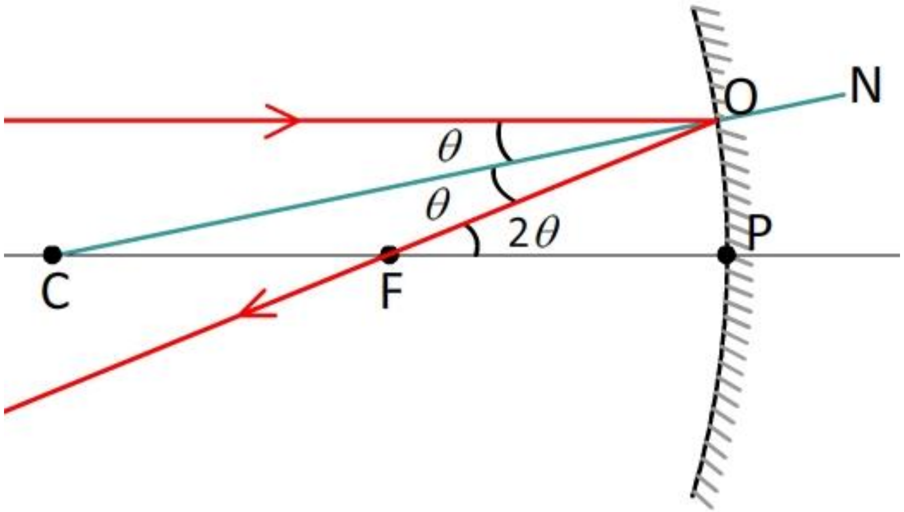


Relation between R and f

Consider a concave mirror of focal length f and radius of curvature R. Let a ray of light , parallel to the principal axis , be incident on the mirror. Upon reflection, the ray passes through focal point (F). Angle of incidence (θ) is equal to angle of reflection (θ).

Considering triangle COF. Exterior angle is equal to the sum of interior opposite angles.

Angle OFP = 2θ — i



Using paraxial ray approximation

$$\tan(2\theta) = \frac{OP}{f} \qquad \tan(\theta) = \frac{OP}{R}$$

For small angles, $\tan(\theta) \approx \theta$

$$2\theta = \frac{OP}{f} \Rightarrow OP = f \times 2\theta$$ — ii

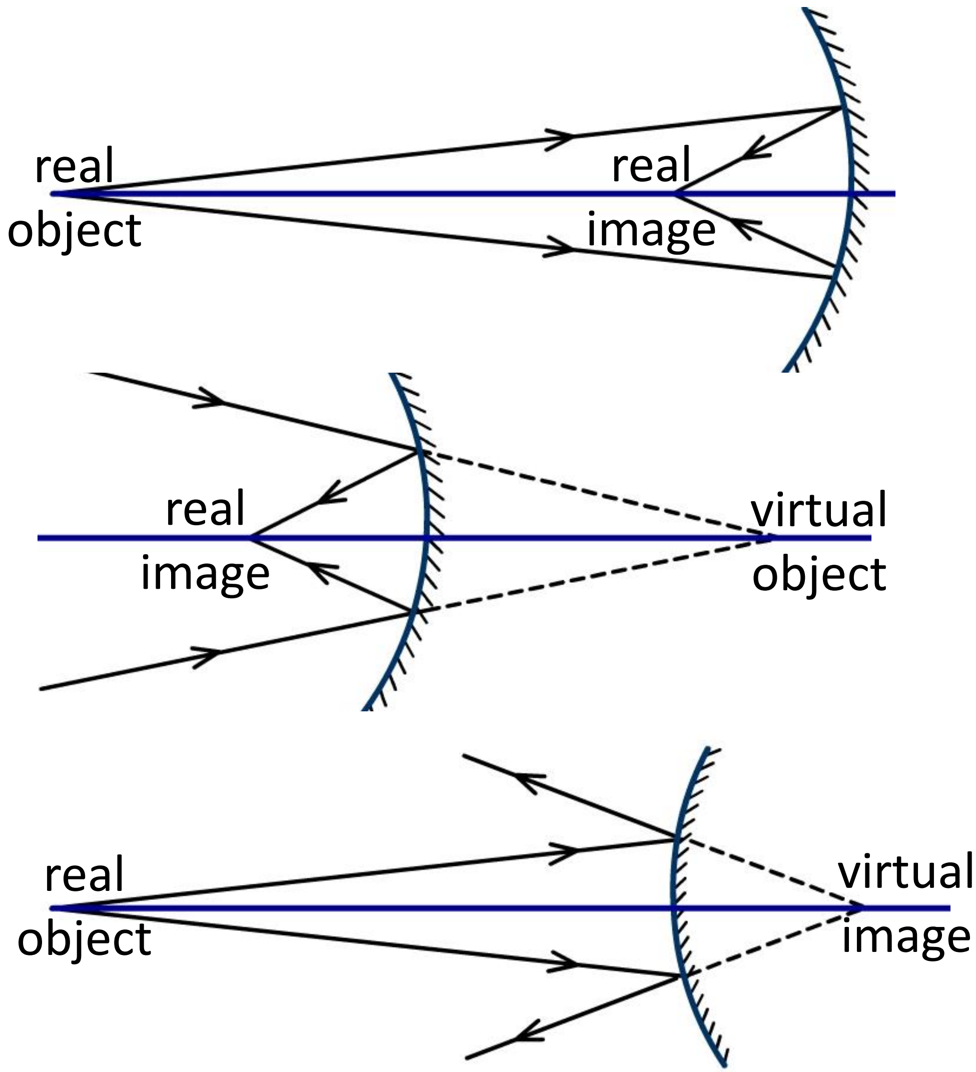
$$\theta = \frac{OP}{R} \Rightarrow OP = R \times \theta$$ — iii

Equating (ii) and (iii)

$$R \times \theta = f \times 2\theta$$

$$\boxed{R = 2f}$$

Real and virtual (objects and images)



Real object : incident rays diverge away from a real object

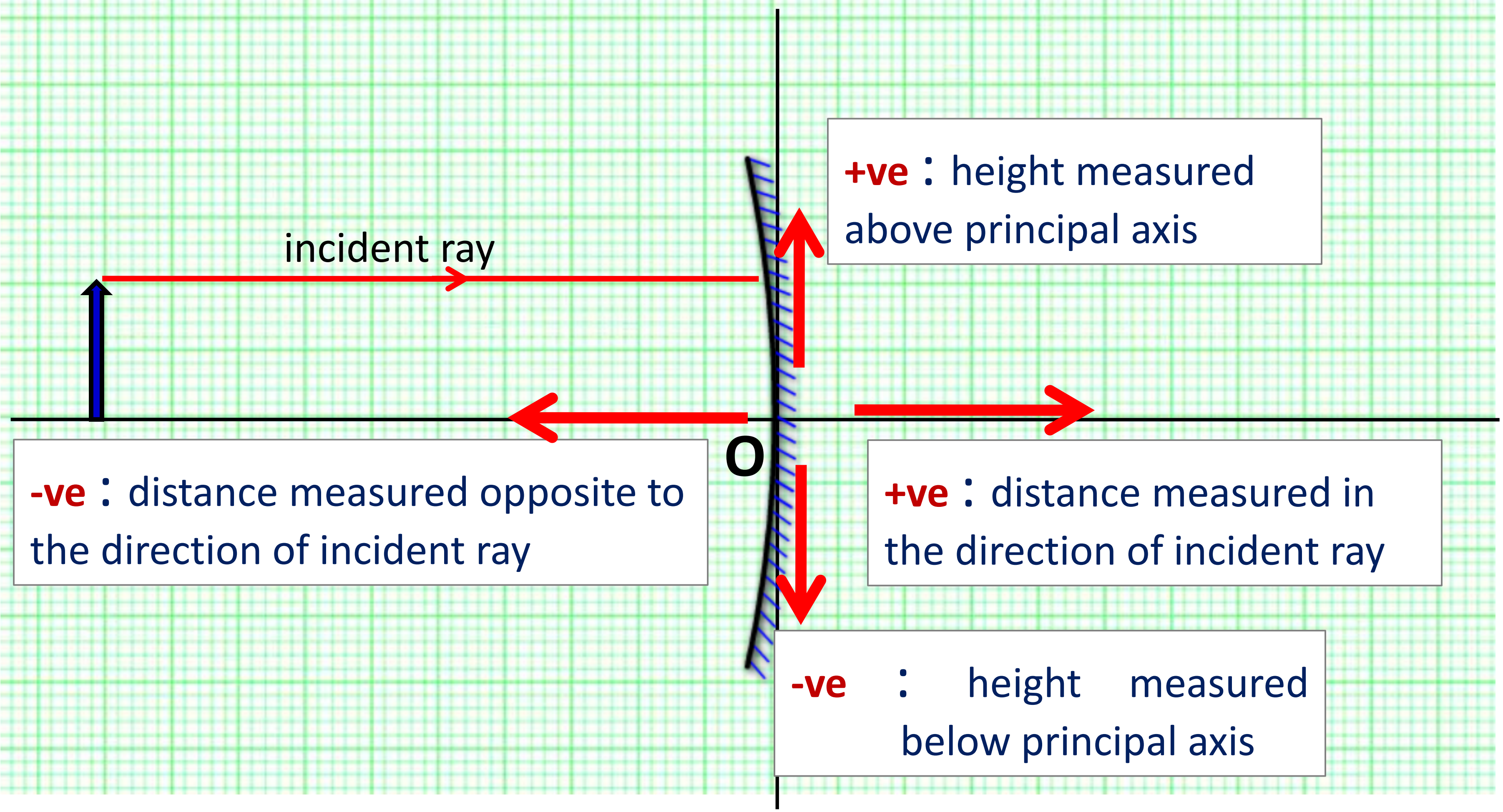
Virtual object : incident rays converge towards a virtual object

Real image : rays converge to a point after reflection

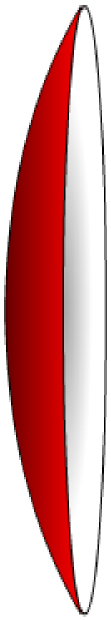
Virtual image : rays diverge away from each other after reflection. Extension of reflected rays converge at a point.

New Cartesian sign convention

Pole is taken as reference for position (origin) and direction of **incident ray** is taken reference (i.e. positive) for direction.

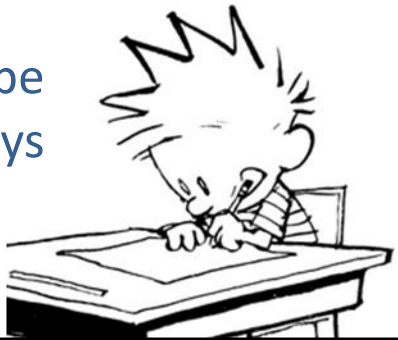


Concave mirror

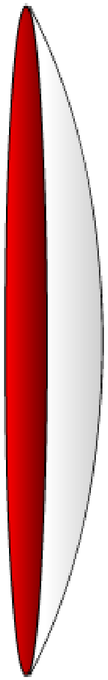


- Concave mirror forms both real and virtual images
- Focal length of a concave mirror is negative.
- Concave mirror always tends to converge a beam of light.
- Real image of a real object is always inverted. It may be enlarged or diminished or of the same size (depending on the object distance)
- Virtual image of a real object is always erect and enlarged.

Note: The image said to be erect or inverted always w.r.t. object



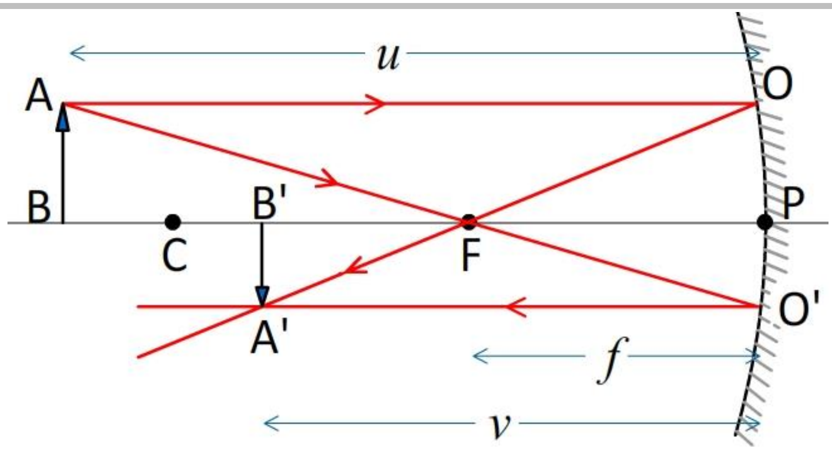
Convex mirror



- Image of a real object is always virtual erect and diminished.
- Focal length of a convex mirror positive.
- Convex mirror always tends to diverge a beam of light.

Mirror formula (derivation)

Consider an object (AB) placed at a distance u from the pole of a concave mirror of focal length f. Real inverted and diminished image (A'B') is formed at a distance v as shown in the figure.



Click here for simulation

Considering similar triangles ABF and FPO'

$$\frac{AB}{BF} = \frac{PO'}{PF}$$

$$\Rightarrow \frac{AB}{PO'} = \frac{BF}{PF}$$

PO' = A'B' therefore

$$\Rightarrow \frac{AB}{A'B'} = \frac{u - f}{f} \quad \text{i}$$

Considering similar triangles A'B'F and FPO

$$\frac{A'B'}{B'F} = \frac{PO}{PF}$$

$$\Rightarrow \frac{PO}{A'B'} = \frac{PF}{B'F}$$

PO = AB therefore

$$\Rightarrow \frac{AB}{A'B'} = \frac{f}{v - f} \quad \text{ii}$$

Equating (i) & (ii)

$$\frac{u - f}{f} = \frac{f}{v - f}$$

$$uv - vf - uf + f^2 = f^2$$

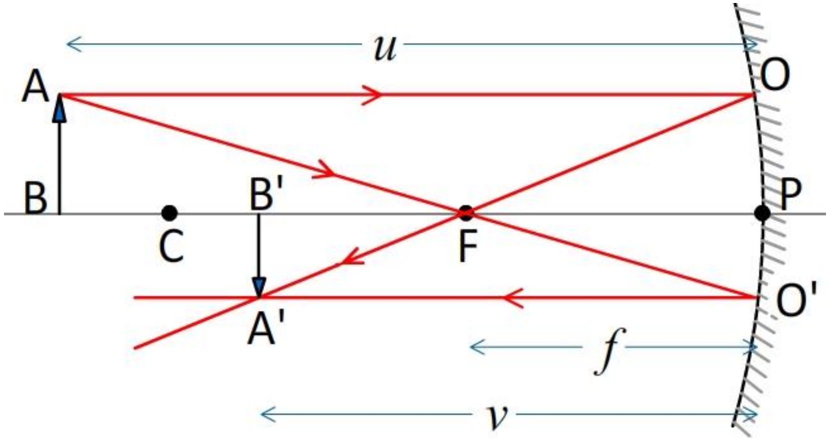
$$uv - vf - uf = 0$$

Dividing by uvf we get

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Mirror formula (derivation) contd.

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$



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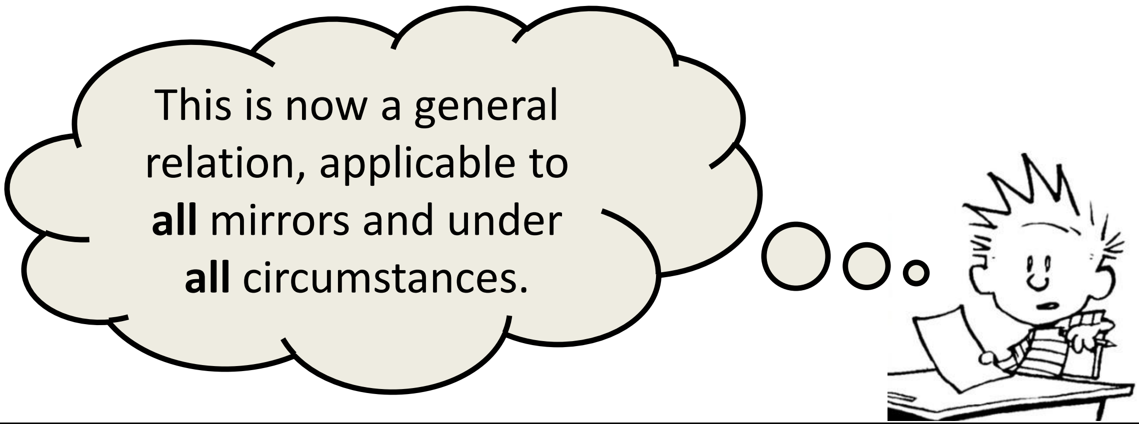
The above relation is a relation specific to the diagram taken as reference. To make is applicable to all situations we generalize it by applying sign convention specific to the reference considered for the derivation.

Incident direction is towards RHS. Therefore focal length (f) , object distance (u) and image distance (v) are all negative as they are measured LHS from the pole. Using this in the above equation we (again) get

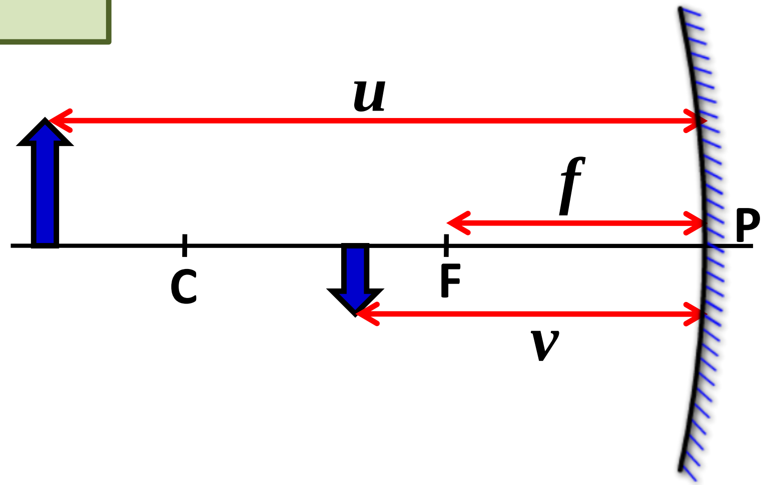
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$



Mirror formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This **general relation** and is applicable to any mirror with **suitable sign convention**



Object distance : u
Image distance : v
Focal length : f

Image formation by a concave mirror

Click here for simulation

Image formation by a convex mirror

Click here for simulation

Lateral magnification

$$\boxed{m = \frac{H}{h}}$$

$$\boxed{m = -\frac{v}{u}}$$

These are **general relations** and are applicable to any mirror with **suitable sign convention**

Object distance : u
Image distance : v
Object height : h
Image height : H

Positive value of magnification implies that the image is **virtual**

Negative value of magnification implies that the image is **real**

If **m > 1** it implies that the image is **magnified**

If **m < 1** it implies that the image is **diminished**

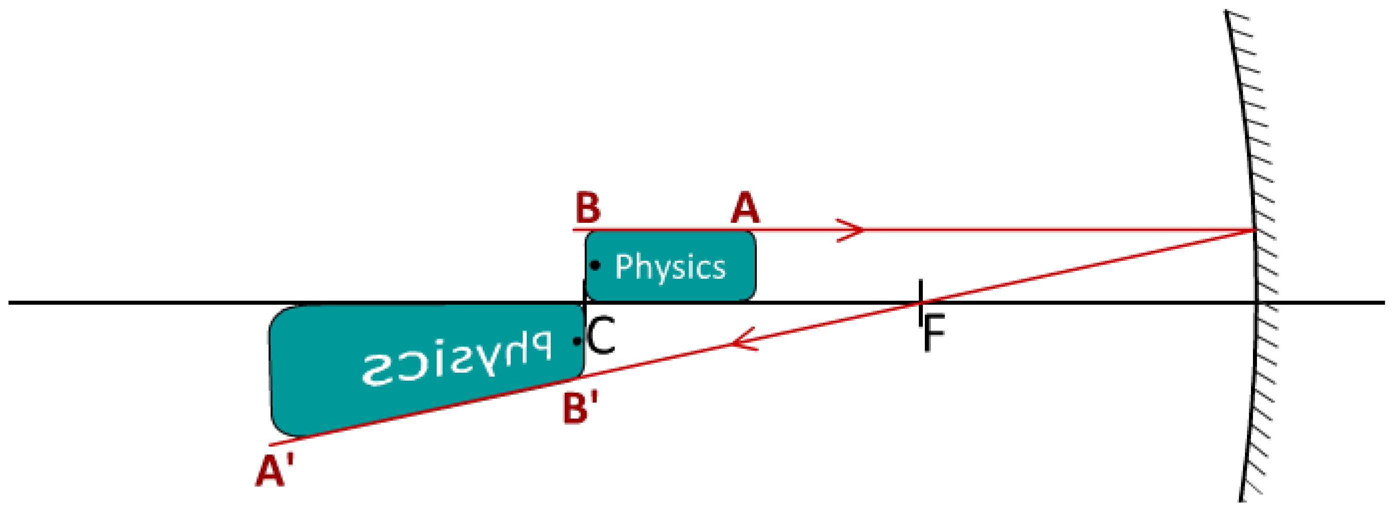
Image formation by a concave mirror

Click here for simulation

Image formation by a convex mirror

Click here for simulation

A mobile placed in front of concave mirror



One end of the mobile (A) is closer to the mirror and therefore its image is (A') is formed farther from the pole. This also results in grater magnification.

The other end of the mobile (B) is at the centre of curvature. Its image (B') is also formed at centre of curvature (with same magnification).

The combined effect of varying object distance results in varying image distance and varying magnification. Image of the mobile is therefore uneven.

Note on problem solving

- Problems in reflection of light from spherical (or plane) mirrors can be solved by using the *general* relations (with suitable *sign conventions* as and when required).
- Quantities given in questions in Boards and in competitive examinations ***will not*** be with sign conventions.
- For reflections from more than one mirror, consider one reflection at a time. Use sign convention for that reflection considering pole as origin and direction of light ray incident on it for positive direction.
- Image formed due to one reflection acts as a object for the next reflection (and so on).